2013

Paper I (80)

Full Marks : 100

Time : 3 hours

Answer any **ten** questions

The questions are of equal value

Candidates are required to give their answers in their own words as far as practicable.

1. Discuss necessary and sufficient condition for uniform convergence.

2. Test the convergency of $\int_0^\infty \frac{xdx}{1+x^4\cos^2(x)}$

3. State and prove Weierstrass M-Test for the uniform convergence of a series.

4. Show that the series $\sum \frac{a_n x^n}{1+x^{2n}}$ converges uniformely for all values of x, if $\sum a_n$ is absolutely convergent.

(*Turn Over*)

5. The Fourier series $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ of $f(x)$ converges uniformly to $f(x)$ at every point of the interval $(0, 2\pi)$ then prove that

$$\frac{1}{\pi}\int_0^{2\pi}[F(x)]^2 dx = \frac{a_0^2}{2}+(a_n^2+b_n^2)$$

6. Explain the inequality $f_{xy}(0,0) \neq f_{yx}(0,0)$ by Schwarz's theorem for the function

$$f(xy) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

7. State and prove implicit function for a function of several variables.

8. Show that the simultaneous limit exist at the origin but the repeated limits donot exist for the functions defined by

$$f(x,y) = \begin{cases} x\sin\left(\frac{1}{y}\right) + y\sin\left(\frac{1}{x}\right), & x,y \neq 0 \\ 0, & xy = 0 \end{cases}$$

9. Show that the function

$$f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & x,y = (0,0) \end{cases}$$

is continuous but not differentiable at (0,0).

10. If $E_i, \forall i = 1,2,....$ are measurable sets then the set $\bigcap_{i=1}^{\infty}(E_i)$ is also measurable.

11. If $\langle E_n \rangle$ is monotonically increasing sequence of measurable sets and $E = \bigcup_{n=1}^{\infty} E_n$ then $m(E) = \lim_{n\to\alpha} m(E_n)$.

12. If E_1 and E_2 are measurable subset of $[a, b]$ then prove that $m(E_1)+m(E_2) = m(E_1 \cup E_2) + m(E_1 \cap E_2)$.

13. Every bounded measurable function f defined on $[a, b]$ is Lebesgue integrable over $[a, b]$.

14. If f is bounded function, Lebesgue integrable an a measurable subset of $[a, b]$ then $|f|$ is also Lebesgue integrable on E and $\left|\int_E f\right| \leq \int_E |f|$.

15. Prove that the function

$$f(x) = \frac{d}{dx}\left(x^2 \sin\left(\frac{1}{x^2}\right)\right) = 2x \sin\left(\frac{1}{x^2}\right) - \frac{2}{x}\cos\left(\frac{1}{x^2}\right)$$

is not Lebesgue integrable over $[0,1]$.
