Total Pages : 4 UM(I)-M(3)

2013

Paper – III
75

Full Marks : 100

Time : 3 hours

Answer any ten questions

The questions are of equal value

Candidates are required to give their answers in their own words as far as practicable.

1. Show that a subset F of a metric space (E, d) is closed iff the limit of every convergent sequence of points of F is again a point of F.

2. Prove that (R^n, d) is a complete metric space. Where $d(x,y)=\sqrt{\sum_{i=1}^{n}(x_i-y_i)^2}$ for two n-tuples x and y.

(*Turn Over*)

3. Let E and F be two metric spaces and $f : E \to F$ be a mapping of E into F. Then show that f is continuous iff inverse of every closed subsets H of F is closed subset of E.

4. Prove that a metric space (X, d) is sequentially compact iff X is complete and totally bounded.

5. Show that if X is a sequentially compact metric space, then any open covering $\{G_\alpha\}$ of X has a Lebesgue number.

6. State and prove Weierstrass Approximation theorem.

7. Prove that $C_0(X, R)$ is a closed subalgebra of $C(X, R)$.

8. Let (X, d) be a metric space. Let T be the family of those subsets G of X which are open with respect to metric d. Then T is a topology on X. Verify.

9. Given a non-empty family S of subsets of a set X. Show that there exists a weakest topology T on X in which all members of S are open sets and S is a subbase for T.

10. Show that the product space of any non-empty class of Hausdorff space is a Hausdorff space.

11. Let $\{X_i\}_{i \in I}$ be a non-empty class of topological spaces. Show that the product-space $X = \prod_{i \in I} X_i$ is compact if and only if each X_i is compact.

12. Show that every metric space is normal.

13. Show that every completely regular space is regular T_1-space.

14. Show that a subspace X of the real line R with usual topology is connected if and only if X is an interval.

15. Write short notes :

(*i*) Homotopy

(*ii*) Fundamental group.
