2013

Paper V
81

Full Marks : 100

Time : 3 hours

Answer any **ten** questions

The questions are of equal value

Candidates are required to give their answers in their own words as far as practicable.

1. If the tangent and the binormal at a point of a curve make angles θ, ϕ respectively with a fixed direction, show that $\dfrac{\sin\theta d\theta}{\sin\phi d\phi} = -\dfrac{\kappa}{\tau}$. 3

2. Show that a curve is uniquely determined, except as to position in space, when its curvature and torsion are given functions of its arc length s. 10

3. Define a Helix. Prove that a curve whose curvature and torsion are in a constant ratio is a helix. 9

8

(*Turn Over*)

4. Derive the condition that the surface $z = f(x, y)$ may be a developable surface. 10

5. State and prove Meunier's Theorem. 10

6. Derive the condition that the two directions given by $Pdu^2 + Qdudv + Rdv^2 = 0$ be at right angles.

7. State and prove Joachimsthal's Theorem.

8. State and prove Euler's theorem on normal curvature. 10

9. Obtain the expressions for E, F, G and L, M, N for the surface of revolution. 10

$$x = u\cos\phi, \quad y = u\sin\phi, \quad z = f(u).$$

10. Obtain Gauss characteristic equation. 9

11. For any surface, prove that $\frac{\partial}{\partial u}(\log H) = l + \mu$ where symbols have their usual meaning.

12. Obtain the torsion of a geodesic as a scalar triple product. 8

13. Show that sum of two tensors is a tensor of the same rank and similar character.

14. Prove that

(i) $\Gamma_{ij,k} + \Gamma_{jk,i} = \frac{\partial g_{ik}}{\partial x^j}$

(ii) $g_{kp}\Gamma^k_{ij} = \Gamma_{ij,p}$.

15. Obtain covariant derivative of a covariant vector.