2013

Paper VI
70

Full Marks : 100

Time : 3 hours

Answer any **ten** questions

The questions are of equal value

Candidates are required to give their answers in their own words as far as practicable.

(10) 1. Discuss the motion of a uniform rod of mass m and length $2a$ which can turn freely about its one fixed end. Suppose that it starts with angular velocity ω from its vertical rest position.

(10) 2. Find the moment of momentum about the origin for a body moving in two dimensions.

3. A uniform rod slides down in a vertical plane whose ends are in contact with two smooth planes, one horizontal and other vertical. If it starts from rest at an angle α with the horizontal, discuss the motion.

4. Derive Euler's geometrical equations.

5. Find the components of angular velocity Ω of a body turning about some instantaneous axis through a fixed point relative to moving axes in three dimensions. Also find the components of angular velocity Ω if the moving axes are fixed in the body.

6. Discuss the motion of a body in three dimensions.

7. A rigid body turns about some fixed point O. If w_1, w_2, w_3 and m_1, m_2, m_3 are the angular velocities and moments of intertia respectively about the principal axes OA, OB, OC; then show that the K.E is given by $\frac{1}{2}\left(m_1 w_1^2 + m_2 w_2^2 + m_3 w_3^2\right)$.

8. Define holonomic and non-holonomic dynamical systems and derive Lagrange equation of motion for a holonomic dynamical system.

9. Prove that the total energy of a conservative holonomic dynamical system is constant.

(Continued)

10. Derive Lagrange equation for small oscillation of a dynamical system.

11. Establish the relations

(i) $\frac{\partial H}{\partial q_r} = -\dot{p}_r$

(ii) $\frac{\partial H}{\partial p_r} = \dot{q}_r$

where symbols have their usual meaning.

12. Define Routhian function. Do you agree that it behaves like both Lagrangian and Hamiltonian functions ? If yes, justify your answer.

13. Using Poisson-bracket condition. Show that the transformation defined as

$$p = \sqrt{2P}\cos Q$$
$$q = \sqrt{2P}\sin Q$$

is canonical.

14. Find a geodesics (shortest distance) on a right circular cylinder.

(Turn Over)

10

15. State and prove the principle of least action.
