Paper VII
90

2013

Full Marks : 100

Time : 3 hours

Answer any **eight** questions

The questions are of equal value

Candidates are required to give their answers in their own words as far as practicable.

1. Define a lattice in two equivalent ways and establish their equivalence.

2. Define a modular lattice and establish a necessary and sufficient condition for a lattice to be modular.

3. Show that every distributive lattice is modular but converse is not true.

4. Let $(X, +, \cdot, ')$ be a finite Boolean algebra. Then show that every element of X can be uniquely expressed as a sum of atoms.

(*Turn Over*)

5. Discuss applications of Boolean algebra to switching networks.

6. Let G be a simple graph with n-vertices, k-components and m-edges. Then prove that

$$(n-k) \le m \le \frac{(n-k)(n-k+1)}{2}$$

7. Show that a connected graph is Eulerian if and only if set of edges can be split up into disjoint cycles.

8. If G is a simple graph with $n(\ge 3)$ vertices, and if $\deg(v)+d(w) \ge n$, for each pair of non-adjacent vertices v and w, then show that G is Hamiltonian.

9. State marriage problem and prove that a necessary and sufficient condition for a solution of the marriage problem is that each set of k-girls collectively knows at least k-boys.

10. If 20 candidates appear in a competition examination, then show that there exist at least two among them, whose roll numbers differ by a multiple of 19.

11. If two states of a finite state machine are equivalent, then prove that their next states are also equivalent.

12. Let M be a finite state machine, with state table given below :

State	f(Input)			g(Output)		
	x	y	z	x	y	z
S_0	S_0	S_3	S_2	0	1	1
S_1	S_1	S_1	S_3	0	0	1
S_2	S_1	S_2	S_3	1	1	0
S_3	S_2	S_3	S_0	1	0	1

Draw the state (Transition) diagram o f M. Starting at S_0, What is output for the input string $xyyzzz$?

13. Design a finite state automaton that excepts those strings over $\{0,1\}$ such that number of zeros is divisible by 3.

14. If L is a set accepted by a non-deterministic finite automaton, then show that there exist a deterministic finite automaton that accepts L.

15. Write short note on Turning machine.
